JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 23, No. 6, November–December 2000

Novel Method for Identification of Aircraft Trajectories in Three-Dimensional Space

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In this paper a novel method for identification of aircraft trajectories in three-dimensional space is considered. The differential equations of the vector field of an aircraft's velocity are presented and analyzed. The inverted approximation method is proposed for estimating the smooth trajectories from the discrete and sparse past position readings of an aircraft in order to identify the state variables and kinematic parameters of an aircraft's movement. The linear and vectorizable procedure as well as its time delay neural network implementation for the purpose of a trajectory recognition is outlined. The performance of this method is assessed in the simulation of maneuvering target interception.

Introduction

THE main purpose of the tracking system for air defense is the estimation of target trajectories in the controlled area and their prediction into the near future. Mathematical modeling of the target tracking process has always been a subject of extensive studies.[1,2]

The aircraft's trajectory is usually recognized by means of a statistical approach to a time series analysis of radar plots. Since the early 1960s, the Kalman filter and its variants have preferably been used for tracking applications.[1−3] According to this approach, the state of the tracked aircraft consists of its position and the time derivatives of displacement. This displacement of an arbitrarily maneuvering aircraft would, in general, have a number of nonzero derivatives. An accurate model of the aircraft motion should include all of these derivatives. Airplanes' motion models, which are currently available in tracking literature, are including terms up to the second derivatives of the aircraft position.[2,3] These tracking techniques are well suited for the recognition of the straight line or moderately curved trajectories, whereas modeling of the modern generation of highly maneuvering aircraft call for better tracking performance than what is provided by acceleration models. The alpha-beta and alpha-beta-gamma trackers or even bank filters with the switching of structure represent classical approaches for solving this problem.[4,5] The reason for the inadequate tracking performance of current models is that higher-order derivatives in the case of very highly maneuverable aircraft are not insignificant, leading to model inaccuracies, when terms only up to aircraft acceleration are included.[3] As a result, they do not easily comply with the physics of real aircraft flight in an aerodynamically dissipating environment. To formulate the essential assumptions for tracking of the tactical aircraft trajectory, a deep analysis of aircraft flights is needed from the point of view of a ground observer (i.e., radar system). Considering the real aircraft's flight in an aerodynamically dissipating environment, it is important to realize that it should be seen as a complex interaction between two main processes: 1) the movement of an aircraft treated as a rigid body with a longitudinal plane of symmetry and having its own variable propulsion (occurred with fuel consumption by the jet's engine and following the aircraft's mass reduction during flight in constant gravitational field), and 2) the dynamic interaction between the airflow and the aircraft's profile resulting in aerodynamic drag and lift forces resisting and balancing the aircraft's movement.[6,7]

On the other hand, the radar tracking system recognizes the arbitrary trajectory of an airplane's movement as an optical flow of a visible particle and its vector field of velocity in three-dimensional space.[8,9] Consequently, if the vector relations between the state variables and kinetic parameters of an aircraft are to be properly identified, the tracking procedures must appreciate additional constraints including the total energy conservation, the rectifiability of trajectory, and the coordinated turns of aircraft in the feasible process of flight. For this purpose each unknown trajectory of an aircraft (i.e., a rectifiable continuous and smooth curve in three-dimensional space) should be considered as a result of a stable process of aircraft flight in an aerodynamically dissipating environment (i.e., it must be seen as the coupled Hamiltonian system of the moving flat and symmetrical rigid body with its own propulsion and adjacent air flow generated around the aircraft's profile).[8]

Apparently, any fast disturbances are not essential for the perfect study of this flow phenomenon, because a relatively long period of past observations (approximately 10 chaotic samples of position readings) is needed. In the tracking practice smooth solutions are preferable so that the noiseless model would be even more appropriate to describe the ideal aircraft's trajectory by means of low-order differential equations in three-dimensional space.[7,9] In recent years, considerable progress has been made in modeling chaotic time series with neural networks. Although neural networks are not a panaceum, they can provide significant advantages over classic techniques and can also be implemented for real-time solutions.[10,11] In this paper mathematical modeling and simulation have been used to develop a new method of recognizing highly maneuverable aircraft trajectories in three-dimensional space. The differential equations of the vector field of an aircraft's velocity are presented and analyzed.

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Received 14 July 1998; revision received 3 March 2000; accepted for publication 12 April 2000.

The inverse approximation (according to Polish AK method) has been adapted for the estimation of smooth trajectories in three-dimensional space.[12–14] An efficient procedure has been developed for the identification of the state variables and kinematic parameters of aircraft movement. The recurrent network of time delay neural network (TDNN) type has been proposed for the estimation of the trajectory in three-dimensional space.[15] Several simulation experiments have been carried out to prove the usefulness of this methodology. For this purpose a pursuit–evasion scenario has been arranged, and the time series of numerical data has been artificially generated to model the successive measurements of aircraft position in three-dimensional space in order to drive the tracking algorithm. Furthermore, modeling errors have been estimated to achieve a sufficient degree of simulation fidelity. The simulation results show that effective tracking of a highly maneuverable aircraft requires implementations of differential filters of the TDNN type [preferably implemented in very large scale of integration technology (VLSI)]. This paper covers the following topics: 1) mathematical modeling of aircraft trajectory in three-dimensional space geometry; 2) discussion of the proposed model; 3) statement of the identification problem; 4) solution to the identification problem; 5) error estimation; 6) application of TDNN to trajectory recognition; 7) simulation experiments; and 8) conclusions.

Mathematical Modeling of Aircraft Trajectory in Three-Dimensional Space

Apparently, the rigid-body movement (i.e., aircraft) is well described by Hamiltonian equations, but for practical purposes of aircraft trajectory tracking it is too complicated a mathematical description.[8]

On the other hand, according to energy conservation, the total fuel consumption during an aircraft's flight along an arbitrary trajectory is determined by the balancing of the energy dissipation (mainly caused by aerodynamic drag) and mechanical energy accumulation (caused by the variation of velocity and altitude of an aircraft's flight). In other words, the energy supplied with the fuel to the system is wholly distributed to the force of the fictive optical flow of the vector velocity field (observed by the radar) of the aircraft.[9] As a result, it may be well modeled as the Bernoulli-like fictive flow of a thin stream of an ideal liquid in three-dimensional space along this trajectory.[8] Consequently, the succeeding past positions of the aircraft along its trajectory can be interpreted as a continuum of visible particles of a fictive optical flow within the observation area of the radar system.

Thus, the aircraft trajectory, which can be recognized by the ground radar tracking system, consists of an arbitrary continuous and smooth curve additionally constrained by the following conditions: 1) in the kinematic sense the aircraft velocity, acceleration, and angular rate vectors at each point of trajectory should together define the Lie algebra in three-dimensional Euclidian space; and 2) in the geometric sense the trajectory should be a geodesic line in this space (i.e., be rectifiable).[8]

To consider all these aspects just discussed, the appropriate frame of reference must be defined, and the aircraft (a flat rigid body with its own propulsion) should be modeled as a single weighted particle with a velocity vector, attached to the weightless lifting plane (in its center of gravity). Next, the movement of the defined airplane is examined in an aerodynamic environment during a finite period of observation $[t_0, t_0 + T_p]$ in a right-handed inertial frame of reference $Oxyz$ (i.e., Cartesian coordinate system). Furthermore, the two additional frames of reference $(Cx'y'z')$ and $(Cx''y''z'')$, which are attached to the moving particle (i.e., center of gravity of airplane), are considered and are defined as follows (see Fig. 1):

1) For the $(Cx'y'z')$ frame the x', y', and z' axes are parallelly oriented to the x, y, and z axes of the inertial frame of reference $(Oxyz)$;

2) For the $(Cx''y''z'')$ frame the x'', y'', and z'' axes have Frenet frame configuration, such that the x'' axis is collinear with the vector of linear velocity of maneuvering aircraft, and it always lies on the strictly tangential surface (of the Frenet frame) to the trajectory of aircraft's movement. The y'' axis is colinear with the vector of centripetal force acting on maneuvering aircraft, and the z'' axis

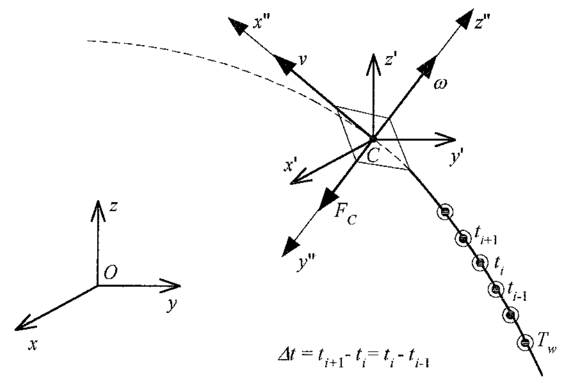

Fig. 1 Configuration of reference frames.

is colinear with the vector of the angular rate of a maneuvering aircraft.

In thus defined frames of reference, the Euler angles (ψ, θ, φ) (i.e., yaw angle, flight-path angle, and bank angle) naturally describe the orientation of the Frenet frame (and the lifting plane of aircraft $Cx''y''$) with respect to the inertial frame of reference $(Oxyz)$. The arbitrary trajectory of a maneuvering aircraft can be now described in the $(Oxyz)$ frame by the following set of equations:

$$x(t + \Delta t) = x(t) + v_x(t) \cdot \Delta t, \qquad y(t + \Delta t) = y(t) + v_y(t) \cdot \Delta t$$
$$z(t + \Delta t) = z(t) + v_z(t) \cdot \Delta t, \qquad \psi(t + \Delta t) = \psi(t) + \omega_z(t) \cdot \Delta t$$
$$\text{for} \qquad t \in [t_0, t_0 + T_p] \tag{1}$$

where T_p is the possible horizon of trajectory prediction and Δt is the time step of update.

Whereas the kinematic relations [consisting of the nonlinear state equations of an aircraft's model of flight in the $(Oxyz)$ frame] take the form

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v_x(t) = v(t) \cdot \cos[\theta(t)] \cdot \sin[\psi(t)]$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = v_y(t) = v(t) \cdot \cos[\theta(t)] \cdot \cos[\psi(t)]$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = v_z(t) = v(t) \cdot \sin[\theta(t)]$$
$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = \omega_z(t) = -\frac{g \cdot \mathrm{tg}[\varphi(t)]}{v(t) \cdot \cos[\theta(t)]} \tag{2}$$

where g is the acceleration as a result of gravity and the geometric relations [defining the natural nonlinear output equations of an aircraft's model of flight in $(Oxyz)$ frame of reference] take the form

$$|v(t)| = \sqrt{v_x^2(t) + v_y^2(t) + v_z^2(t)}, \qquad \theta(t) = \mathrm{arctg}\frac{v_z(t)}{\sqrt{v_x^2(t) + v_y^2(t)}}$$
$$\psi(t) = \mathrm{arctg}\frac{v_x(t)}{v_y(t)}, \qquad \varphi(t) = \mathrm{arctg}\frac{v(t) \cdot \omega_z(t)}{\dot{v}(t)}$$
$$\text{for} \qquad \dot{v}(t) \neq 0, \qquad v_x(t) \neq 0, \qquad v_y(t) \neq 0 \tag{3}$$

and

$$|\dot{v}(t)| = \sqrt{\dot{v}_x^2(t) + \dot{v}_y^2(t) + \dot{v}_z^2(t)} \tag{3a}$$

The set of Eqs. (1–3) fully defines the trajectory of the aircraft's flight. They look like a model of a differentially flat system, which can be well used for aircraft trajectory prediction and generation.[6] Strictly speaking, they could be considered as the differentially flat system if it were possible to express the angular rate $\omega_z(t)$ by means of velocity components $\{v_x(t), v_y(t)\}$ and their higher-order time derivatives.

The answer to the question "How is it done?" is revealed further in the paper.

Discussion on the Mathematical Model

According to Euler theory, the group of revolutions in three-dimensional Euclidean space (provided with the left-invariant Riemannian metric) is sufficient to express an arbitrary movement of a rigid body along a geodesic line.[8]

The Euler theory is true not only for rigid-body movement, but it can also be considered in hydrodynamics of ideal liquid flows. For this purpose a group of transformations should consist of a group of volume-conserving diffeomorphisms in flow area. In this case the least action principle means that the movement of ideal liquid takes place along the geodesic line of a right invariantly defined metric (i.e., kinetic energy).[8] Accordingly, the Euler equations of ideal liquid flow in hydrodynamics are analogous to Euler equations of rigid-body movement and especially for aircraft movement in an aerodynamically dissipating environment, where the energy conservation law insists on the equalization between right-invariant metric (for airflow around an aircraft) and left-invariant metric (for aircraft movement). As a result, the trajectory of aircraft movement in the finite area of three-dimensional space can be presented (for tracking purposes) by means of Bernoulli-like partial differential equations of fictive flow of ideal liquid (e.g., optical flow of velocity vector field of aircraft).[8,9] Practically, in this case, the area of trajectory observation can be organized as a moving window (i.e., a three-dimensional shift register with the time width of T_w) being supplied serially (with the time spacing of Δt) with the data of succeeding three-dimensional position readings of the aircraft along its trajectory (including the actual position and the number of past positions, see Fig. 1).

The set of kinematic equations (2) and (3) can be compared (at the front of flow area for the actual time instant) with the equations of the flow of the velocity vector field considered in the finite period of past trajectory observations as follows:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v_x(x, y, z, \tau)|_{\tau=0}, \qquad \frac{\mathrm{d}y}{\mathrm{d}t} = v_y(x, y, z, \tau)|_{\tau=0}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = v_z(x, y, z, \tau)|_{\tau=0}$$

$$\begin{aligned}
\dot{v}_x(t) &= \frac{\partial v_x}{\partial \tau|_{\tau=0}} - 2\omega_z(x, y, z, \tau)|_{\tau=0} \cdot v_y(x, y, z, \tau)|_{\tau=0} \\
&\quad + 2\omega_y(x, y, z, \tau)|_{\tau=0} \cdot v_z(x, y, z, \tau)|_{\tau=0} \\
\dot{v}_y(t) &= \frac{\partial v_y}{\partial \tau|_{\tau=0}} + 2\omega_z(x, y, z, \tau)|_{\tau=0} \cdot v_x(x, y, z, \tau)|_{\tau=0} \\
&\quad - 2\omega_x(x, y, z, \tau)|_{\tau=0} \cdot v_z(x, y, z, \tau)|_{\tau=0} \\
\dot{v}_z(t) &= \frac{\partial v_z}{\partial \tau|_{\tau=0}} + g - 2\omega_y(x, y, z, \tau)|_{\tau=0} \cdot v_x(x, y, z, \tau)|_{\tau=0} \\
&\quad + 2\omega_x(x, y, z, \tau)|_{\tau=0} \cdot v_y(x, y, z, \tau)|_{\tau=0}
\end{aligned}$$

$$\text{for} \quad x(t, \tau) = x(t - \tau), \qquad y(t, \tau) = y(t - \tau),$$

$$z(t, \tau) = z(t - \tau), \qquad \tau \in [0, T_w] \tag{4}$$

and

$$2\boldsymbol{\omega} = \mathrm{rot}\boldsymbol{v} \tag{5}$$

where T_w is the width of the time moving window for trajectory observations.

Equation (4) simply describes both the rotational and translational flow of the vector field of the aircraft's velocity for the past, whereas the kinematic Eqs. (1–3) show the same for the present and near future time instant.

Nevertheless, the main advantage of Eqs. (4) and (5) lies in the fact that they suggest how to identify the angular rate $\boldsymbol{\omega}(t)$ of aircraft during its movement along the curvilinear trajectory. For this purpose we assume that the angular rate $\boldsymbol{\omega}(t)$ should be expressed by means of the following equations:

$$\begin{aligned}
\omega_x(t) &= \tfrac{1}{2}[\dot{v}_z(t)/v_y(t) - \dot{v}_y(t)/v_z(t)] \\
\omega_y(t) &= \tfrac{1}{2}[\dot{v}_x(t)/v_z(t) - \dot{v}_z(t)/v_x(t)] \\
\omega_z(t) &= \tfrac{1}{2}[\dot{v}_y(t)/v_x(t) - \dot{v}_x(t)/v_y(t)]
\end{aligned} \tag{6}$$

and

$$|\omega(t)| = \sqrt{\omega_x^2(t) + \omega_y^2(t) + \omega_z^2(t)} \tag{6a}$$

This formula (6) can be used in Eqs. (1–3) to transform them to the differentially flat system.

If the values $\{v_x(t), v_y(t), v_z(t)\}$ in Eqs. (6) are to appear as zero in any time instant, then the appropriate higher-order derivatives of the aircraft's velocity should be used in relations (6) according to the transformation formula similar to de L'Hôspital rule.

An important property of flat systems is that we can find a set of outputs (equal in number to the number of inputs) so that we can express all states and inputs in terms of those outputs and their derivatives.[6] As follows from Eqs. (1–3) and (6), an arbitrary trajectory of aircraft movement can be easily recognized if the time derivatives of aircraft position can be computed with sufficient accuracy. However, the system equations (1–3) and (6) are continuous, whereas the position readings used in the computation are discrete samples in time.

Statement of the Identification Problem

The problems to be solved are the following:

1) How to estimate the continuous and smooth trajectory of an aircraft from the ground radar plots of past position readings?

2) How to derive the vector field of aircraft velocity and its higher order derivatives for prediction purposes of the aircraft future positions?

To answer this question, let us consider the possibility of numerical differentiation (sufficiently exact) of the smooth approximation of an arbitrary function $y = f(t)$ depending parametrically on time only.

Suppose that in discrete time instants $\{t_i; i = 0, 1, 2, \ldots, n\}$ equispaced with separation Δt in the finite period of time $[0, T]$ the output variable y was measured and took the discrete values y_i for $i = 0, 1, 2, \ldots, n$ with the errors of measurements being considered inessential in this procedure. Now, with the set of data points $\{t_i, y_i; i = 0, 1, 2, \ldots, n\}$ find the approximation $y^* = f^*(t)$ for a real unknown continuous function $y = f(t)$ in such a way that the fitting $y^* = f^*(t)$ would be close to $y = f(t)$ in the sense of L_1 norm both for function and its derivative.

This aim of function approximation can be equivalently expressed by means of the following set of constraining conditions:

$$y_i^* = f^*(t_i) \approx f(t_i) = y_i$$

$$\varphi^*(t_i) = \varphi_i^* = \frac{\mathrm{d}f^*(\tau)}{\mathrm{d}\tau|_{\tau=t_i}} \approx \frac{\mathrm{d}f(\tau)}{\mathrm{d}\tau|_{\tau=t_i}} = \varphi(t_i)$$

$$\text{for} \quad i = 0, 1, 2, \ldots, n \tag{7}$$

and the solution to the task can be found by means of minimization of performance index Q, defined as

$$\min_{f^*} Q = \min_{f^*} \int_0^T \left\{ |f(\tau) - f^*(\tau)| + \left| \frac{\mathrm{d}}{\mathrm{d}\tau} f(\tau) - \frac{\mathrm{d}}{\mathrm{d}\tau} f^*(\tau) \right| \right\} \mathrm{d}\tau \tag{7a}$$

where $f^*(\tau)$ belongs to the class $C^1(0, T)$ of continuously differentiable functions. The performance index [Eq. (7a)] is not differentiable in its global minimum, and the solution to this problem could not be obtained from a variational principle. However, if the following boundary conditions $\{f^*(0) = f(0)\}$ and $\{f^*(T) = f(T)\}$ are fulfilled, then the performance index Q is nonnegatively defined for all arbitrarily selected functions $f^*(\tau) \approx f(\tau)$ for $\tau \in (0, T)$, and the relation $Q \geq 0$ is always valid.

Consequently, the minimal value of Q is to be zero if and only if $f^*(\tau) \equiv f(\tau)$ for $\tau \in [0, T]$. This trivial statement means that each smooth function $y = f(t)$ is the best approximation for itself in the finite domain $t \in [0, T]$.

Nevertheless, this observation does not indicate what form of the function $y = f(t)$ really is. It leads, however, to the fundamental conclusion that the structure of optimal solution $f^*(\tau)$ must be consistent with the one-to-one transformation, and the approximating procedure should define diffeomorphism. Because the particular form of a function $y = f(t)$ is actually unknown, all of the efforts should be focused on the suitable fitting of the function $f^*(t)$ to the experimental data in order to achieve the needed precision (i.e., $Q \leq \varepsilon$) with preferably the lowest cost of operation, i.e., simple computation, saving the computing time and memory resources, as well as assigning the robustness to noise. This nontrivial problem solved in 1972 by A. Korgul by inverted approximation originally developed for modeling and simulation of sugar extraction.[12–14]

Solution to the Identification Problem

Each continuous function $f(t)$ can be considered as a solution to a related but unknown differential equation. Thus, for each final and bounded interval of support $t \in [0, T]$, we can write

$$f(t) = \int_0^t \frac{\mathrm{d}}{\mathrm{d}\tau} f(\tau)\,\mathrm{d}\tau + f(0)$$

or

$$f(t) = f(T) - \int_t^T \frac{\mathrm{d}}{\mathrm{d}\tau} f(\tau)\,\mathrm{d}\tau \tag{8}$$

The relations (8) can also be combined to express function $y^* = f^*(t)$ with the skew-symmetric formula of unitary integration, as follows:

$$y_i^* = f^*(t_i) = \frac{1}{2}[f(0) + f(T)] + \frac{1}{2}\left[\int_0^{t_i} \frac{\mathrm{d}}{\mathrm{d}\tau} f^*(\tau)\,\mathrm{d}\tau - \int_{t_i}^T \frac{\mathrm{d}}{\mathrm{d}\tau} f^*(\tau)\,\mathrm{d}\tau\right]$$
$$\text{for} \quad i = 0, 1, 2, \ldots, n, \qquad \tau \in [0, T] \tag{9}$$

where the unknown derivative $(\mathrm{d}/\mathrm{d}\tau) f^*(\tau) = \varphi^*(\tau)$ is approached with the local basis functions $P_j(\tau)$ according to the formula

$$\varphi^*(\tau) = \sum_{j=0}^{n} \varphi_j^* \cdot P_j(\tau) \qquad \text{for} \qquad j = 0, 1, 2, \ldots, n \tag{10}$$

Local basis functions $P_j(\tau)$ for this purpose are recommended in the form of piecewise second-degree Lagrange polynomials.

We assumed additionally that the value of derivative φ_0^* at the left boundary point is defined in the form of

$$\varphi_0^* = \frac{f(-\Delta t) - f(0)}{\Delta t} \quad \text{or} \quad \varphi_0^* = \frac{f(-\Delta t) - f^*(\Delta t)}{2\Delta t} \tag{11}$$

where $\Delta t = T/n$ and n is the even natural number and $f(-\Delta t)$ is an additional output variable obtained as the retarded value of already measured output $f(0)$.

Substituting Eqs. (10) and (11) into (9), we derive

$$\left[2\boldsymbol{y}^* - \boldsymbol{f}(0) - \boldsymbol{f}(T) - \varphi_0^* \cdot \boldsymbol{a}_0\right]_{n\times 1} = [A - B]_{n\times n} \cdot [\boldsymbol{\varphi}^*]_{n\times 1} \tag{12}$$

where $\boldsymbol{y}^*$ is the column vector of output data, $\boldsymbol{f}(0)$ and $\boldsymbol{f}(T)$ are the column vectors of boundary values of the output data, and $\boldsymbol{a}_0$ is the column vector of initial parameters included in the left-hand side of formula (12), which have the form

$$\boldsymbol{y}^* = \left[y_1^*, y_2^*, \ldots, f(T)\right]^T, \qquad \boldsymbol{f}(0) = [f(0), f(0), \ldots, f(0)]^T$$
$$\boldsymbol{f}(T) = [f(T), f(T), \ldots, f(T)]^T, \qquad \boldsymbol{a}_0 = [a_{10}, a_{20}, \ldots, a_{20}]^T \tag{12a}$$

whereas the column vector of estimated gradients of output data $\boldsymbol{\varphi}^*$ has the form $\boldsymbol{\varphi}^* = [\varphi_1^*, \varphi_2^*, \ldots, \varphi_n^*]^T$. The values of initial parameters a_{10} and a_{20}, consisting of the entry of the column vector a_0, are computed according to the formulas

$$a_{10} = \int_0^{t_1} P_0(\tau)\,\mathrm{d}\tau - \int_{t_1}^{t_2} P_0(\tau)\,\mathrm{d}\tau, \qquad a_{20} = \int_0^{t_2} P_0(\tau)\,\mathrm{d}\tau \tag{12b}$$

and the entries of matrices $[\boldsymbol{A}]$ and $[\boldsymbol{B}]$ in formula (12) are defined as the following values of integrals:

$$a_{ij} = \int_0^{t_i} P_j(\tau)\,\mathrm{d}\tau, \qquad b_{ij} = \int_{t_i}^{T} P_j(\tau)\,\mathrm{d}\tau \qquad \text{for} \quad i, j = 1, 2, \ldots, n \tag{12c}$$

Equation (12) discloses the matrix formula coupling of the estimated output data and their derivatives in finite interval of time $[0, T]$ (e.g., the well-known moving/sliding time window applied usually for time series analysis).

For the proposed parabolical basis functions $P_j(\tau)$, the matrix $[A - B]$ is nonsingular (i.e., $\mathrm{Det}[A - B] \neq 0$), and we can invert Eq. (12), thus obtaining

$$\varphi_0^* = \frac{f(-\Delta t) - f(0)}{\Delta t} \qquad \text{or} \qquad \varphi_0^* = \frac{f(-\Delta t) - f^*(\Delta t)}{2\Delta t}$$
$$[\boldsymbol{\varphi}^*]_{(n\times 1)} = [C]_{(n\times n)} \cdot \left[2\boldsymbol{y}^* - \boldsymbol{f}(0) - \boldsymbol{f}(T) - \varphi_0^* \cdot \boldsymbol{a}_0\right]_{(n\times 1)} \tag{13}$$

where $[C]_{(n\times n)} = [A - B]^{-1}_{n\times n}$ and φ_0^* can also be defined in a different way, more sophisticated and more useful for particular purposes. Through Eq. (13) I have shown how to approximately differentiate the unknown function $y = f(t)$ because it defines the values of derivatives φ_j^* of the approaching function $y^* = f^*(t)$ in the nodal points t_i, $i = 0, 1, 2, \ldots, n$.

This result seems to be essential for the philosophy of quantum computation because it can accept as the input not one number but a coherent superposition of many different numbers and subsequently can perform a computation (a sequence of unitary operations) on all of these numbers simultaneously. This can be viewed as a massive parallel computation or quantum parallelism.[16–18]

Error Estimation

The quality of the differentiating procedure depends on the accuracy of approximation of gradients. To estimate the approximation error, we transform relation (9) as follows:

$$2y_i - f(0) - f(T) = \sum_{j=0}^{n} \varphi_j^* \left[\int_0^{t_i} P_j(\tau)\,\mathrm{d}\tau - \int_{t_i}^{T} P_j(\tau)\,\mathrm{d}\tau\right] + R_i \tag{14}$$

where the residuum R_i in formula (14) takes the form:

$$R_i = \int_0^{t_i} \left[\varphi(\tau) - \sum_{j=0}^{n} \varphi_j^* \cdot P_j(\tau)\right] \mathrm{d}\tau - \int_{t_i}^{T} \left[\varphi(\tau) - \sum_{j=0}^{n} \varphi_j^* \cdot P_j(\tau)\right] \mathrm{d}\tau \tag{14a}$$

Deriving the absolute values for both sides of formula (14a) and using the well-known rules for estimation of the absolute values of integrals and their linear combinations, we obtain the following relation:

$$|R_i| \leq \int_0^{T} \left|\varphi(\tau) - \sum_{j=0}^{n} \varphi_j^* \cdot P_j(\tau)\right| \mathrm{d}\tau \tag{15}$$

Because the basis functions $P_j(\tau)$ are selected as parabola curves, the Simpson scheme can be used to estimate the value of residuum R_i, thus giving the result

$$|R_i| \leq M_5 \cdot T \cdot \Delta t^4 \big/ 180 \tag{16}$$

where the values M_5 and Δt in formula (16) are defined as follows:

$$M_5 = \max_{[0,T]} \left| \frac{d^5}{d\tau^5} f(\tau) \right|, \qquad \Delta t = \frac{T}{n}$$

Having estimation (16) and using Eq. (14a), we derive the following formula for absolute accuracy of the derivatives' approximation:

$$\left| \left. \frac{dy}{d\tau} \right|_{\tau = t_i} - \varphi_i^* \right| \leq \frac{|R_i|}{\sum_{j=0}^{n} |a_{ij} - b_{ij}|} \tag{17}$$

and for the unknown function

$$\left| y_i - f_i^* \right| \leq |R_i|/2 \qquad \text{for} \qquad i = 0, 1, 2, \ldots, n \tag{18}$$

Which means that the proposed method of approximation can secure globally limited error of the function estimation and its derivative. The error depends on the grid of time digitizing in the considered time interval $[0, T]$, as well as on the kind of local basis functions being used for fitting the derivatives of the output variable. If the fitted function $y = f(t)$ is to be a polynomial up to the fourth degree, the presented method is exact, as $\{M_5 \equiv 0\}$.

The convergence rate, the accuracy of setting, and the speed of setting are guaranteed, as the absolute approximation error of the fitted function and its derivative are globally limited according to Eqs. (16–18).

In conclusion, the proposed method is precise enough to differentiate an arbitrary unknown time series of data (being currently collected on-line in the moving time window $[0, T]$), and it can be easily used for trajectory tracking purposes.

Application of TDNN for Trajectory Recognition

Popular tracking techniques are well suited for recognition of the straight line or moderately curved trajectories only, and the modeling of the aircraft's high maneuvers needs to use bank filters or more sophisticated alpha-beta-gamma trackers or even filters with the switching of the structure[2–4]; the analysis and design methods based upon knowledge about phase-space dynamics of nonlinear and dissipative chaotic systems seem to be the most promising tool for this purpose.[10] Thus the time series of radar plots can be treated rather as a chaotic time series of measured data, where the real unknown value is disturbed with the equivalent non-Gaussian or non-Markovian stochastic process (e.g., including also spikes and outliers). In this case the full state vector of a chaotic dissipative system can be easily reconstructed and tracked using only the number of the past position readings of the scalar components (coordinates) of the observed state variables (dependent parametrically on time only). For this purpose each unknown trajectory of an aircraft should be considered as a stable attractor of the dissipative process of aircraft flight in aerodynamic environment. In recent years considerable progress has been made in modeling chaotic time series with neural networks, and the results of these investigations can also be implemented for real-time solutions.[11]

Using the Takens theorem, we can see that for aircraft trajectory recognition (or even its prediction) with the neural network it is enough to know $(2m + 1)$ past position readings.[19] Number m defines the quantity of state equations (2), and in our case $m = 4$. Now we can develop the moving window in the form of generalized tapped delay line and organize it as a linear memory structure (comprising eight memory cells only) with the current updating of the contents of cells. This window is supplied serially on-line with the data of succeeding position readings of aircraft, and the data from this window (i.e., the whole stream of memorized data) are parallely supplied to the inputs of perceptron structure (see Fig. 2).

The perceptron was generally organized according to Eq. (13), but for convenience it was replaced by a more visible and useful formulation of the velocity vector flow in phase space. Namely, the simultaneous collecting and differentiating of the time series data of positions readings (in the moving window $[0, T]$) of each

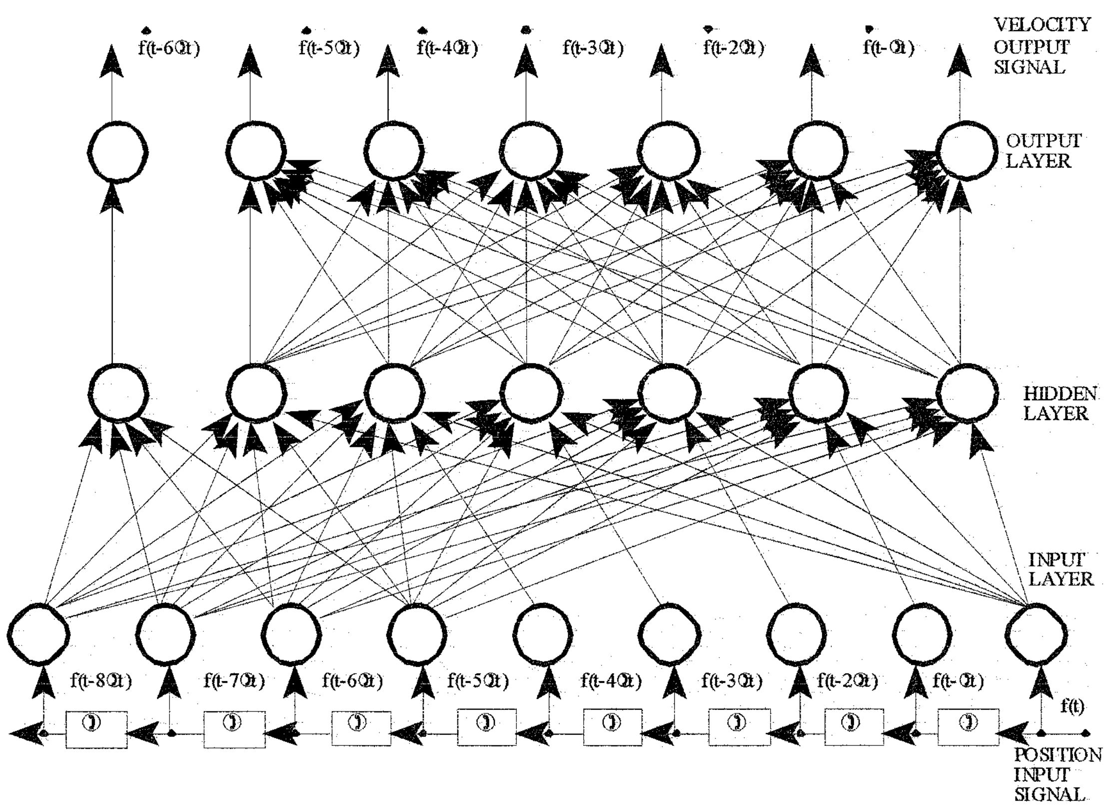

Fig. 2 Time delay neural network for differentiating radar plots (one-dimensional case only).

coordinate (x, y, and z) can be presented according to the following formula:

$$\begin{bmatrix} \dot f(t-6\Delta t) \\ \dot f(t-5\Delta t) \\ \dot f(t-4\Delta t) \\ \dot f(t-3\Delta t) \\ \dot f(t-2\Delta t) \\ \dot f(t-\Delta t) \\ \dot f(t) \end{bmatrix} = \frac{1}{\Delta t}[\boldsymbol{E}]\cdot[\boldsymbol{D}] \begin{bmatrix} f(t-8\Delta t) \\ f(t-7\Delta t) \\ f(t-6\Delta t) \\ f(t-5\Delta t) \\ f(t-4\Delta t) \\ f(t-3\Delta t) \\ f(t-2\Delta t) \\ f(t-\Delta t) \\ f(t) \end{bmatrix} \tag{19}$$

In Eq. (19) matrix $[\boldsymbol{E}]$ takes the form

$$[\boldsymbol{E}] = \begin{bmatrix} e_{00} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} \\ 0 & e_{21} & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} \\ 0 & e_{31} & e_{32} & e_{33} & e_{34} & e_{35} & e_{36} \\ 0 & e_{41} & e_{42} & e_{43} & e_{44} & e_{45} & e_{46} \\ 0 & e_{51} & e_{52} & e_{53} & e_{54} & e_{55} & e_{56} \\ 0 & e_{61} & e_{62} & e_{63} & e_{64} & e_{65} & e_{66} \end{bmatrix} \tag{19a}$$

and matrix $[\boldsymbol{D}]$ takes the form

$$[\boldsymbol{D}] = \begin{bmatrix} d_{01} & d_{02} & d_{03} & d_{04} & 0 & 0 & 0 & 0 & 0 \\ d_{11} & d_{12} & d_{13} & d_{14} & 0 & 0 & 0 & 0 & d_{19} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & 0 & 0 & 0 & d_{29} \\ d_{31} & d_{32} & d_{33} & d_{34} & 0 & d_{36} & 0 & 0 & d_{39} \\ d_{41} & d_{42} & d_{43} & d_{44} & 0 & 0 & d_{47} & 0 & d_{49} \\ d_{51} & d_{52} & d_{53} & d_{54} & 0 & 0 & 0 & d_{58} & d_{59} \\ d_{61} & d_{62} & d_{63} & d_{64} & 0 & 0 & 0 & 0 & d_{69} \end{bmatrix} \tag{19b}$$

If the higher-order derivatives were needed (e.g., $\dddot f$), the additional values of $\dot f(t-8\Delta t)$ and $\dot f(t-7\Delta t)$ should be defined as an appropriately retarded value of $\dot f(t-6\Delta t)$ to adapt formula (19) for further computations.

The moving window and adjacent perceptron structure for the differentiation of measured data can be presented schematically by means of the TDNN graph, as outlined in Fig. 2.

This structure was successfully used for on-line derivation of the following vector fields along an identified trajectory: 1) linear velocity of aircraft $\{v_x(t), v_y(t), v_z(t)\}$; 2) linear acceleration of aircraft $\{\dot v_x(t), \dot v_y(t), \dot v_z(t)\}$; 3) jerk of aircraft $\{\ddot v_x(t), \ddot v_y(t), \ddot v_z(t)\}$; 4) angular rate of aircraft $\{\omega_x(t), \omega_y(t), \omega_z(t)\}$; and 5) angular acceleration of aircraft $\{\dot\omega_x(t), \dot\omega_y(t), \dot\omega_z(t)\}$. The on-line identification of the preceding vector fields allows the recognition of the arbitrary trajectory of the maneuvering aircraft and, using Eqs. (1–3) and (6), the prediction of near future positions precisely enough. This was also used for trajectory generation during the simulation experiment developed to prove the efficiency of this methodology.

Furthermore, this method allows the identification of the aircraft's attitude and attitude rate using a vector of observation of the aircraft's positions only. The angular acceleration is identified, without the need to use of the typically uncertain aircraft dynamic model. It means that gyro-less attitude and attitude-rate estimation can be realized on numerical way only.

Simulation Experiment

Numerical simulation was used to prove the proposed method of trajectory recognition and generation. For this purpose the pursuit scenario was modeled. The time series of numerical data was artificially generated by means of kinematic equations (1–3) to imitate the successive measurements of aircraft position in three-dimensional space. These data were tracked on-line according to procedure (19), which was used as a basic task in target interception modeling.[15] A model of target interception was used, and several simulation experiments were made to obtain the following results.

The four pursuit scenarios were simulated numerically and are shown graphically on the (x, y) plane of Figs. 3–6, respectively. In the preceding scenarios the air target T realizes his maneuvering trajectory outlined as a dotted line. Simultaneously, the fighter F patrols his zone, realizing his own loop-wise trajectory, without any message about the target up to the time moment t_z. In this moment the fighter F got the order (from the ground command control center, i.e., radar tracking and guiding system) to pursue the target T. The fighter F is turned to the target T, in the first step, and next it is guided to the target T with the appropriate homing law. The straight lines in Figs. 3–6, connecting the simultaneous positions of the target T and the fighter F, are the lines of sight, and they characterize the evolution of these situations. In particular, the maneuvering target interception according to long-range proportional navigation was presented in Fig. 3, where the early allocation of the target T took place (i.e., in the time moment $t_z = 200$ s). The similar example of guidance of long-range proportional navigation was illustrated in Fig. 4, but in this case the target allocation took place much later (i.e.,

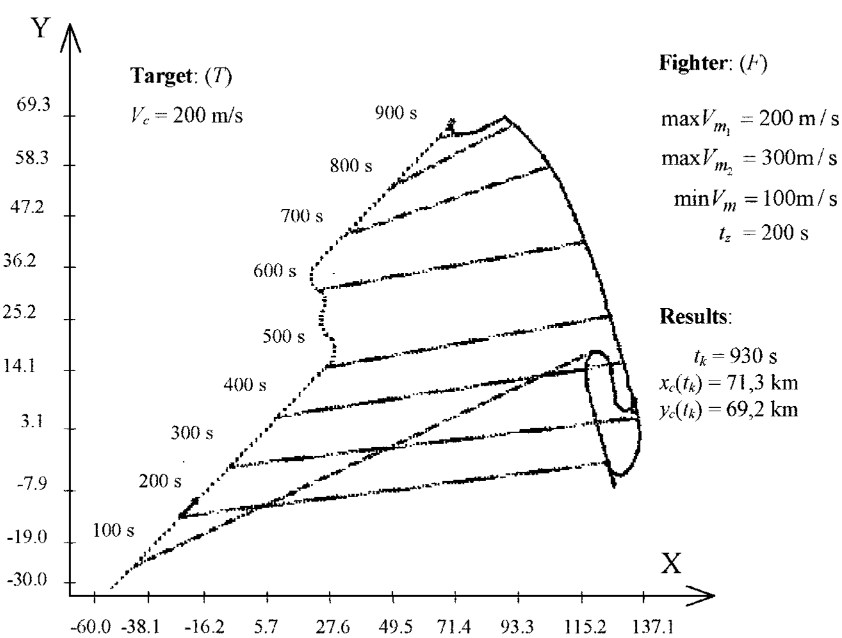

Fig. 3 Target-interception according to long-range proportional navigation guidance.

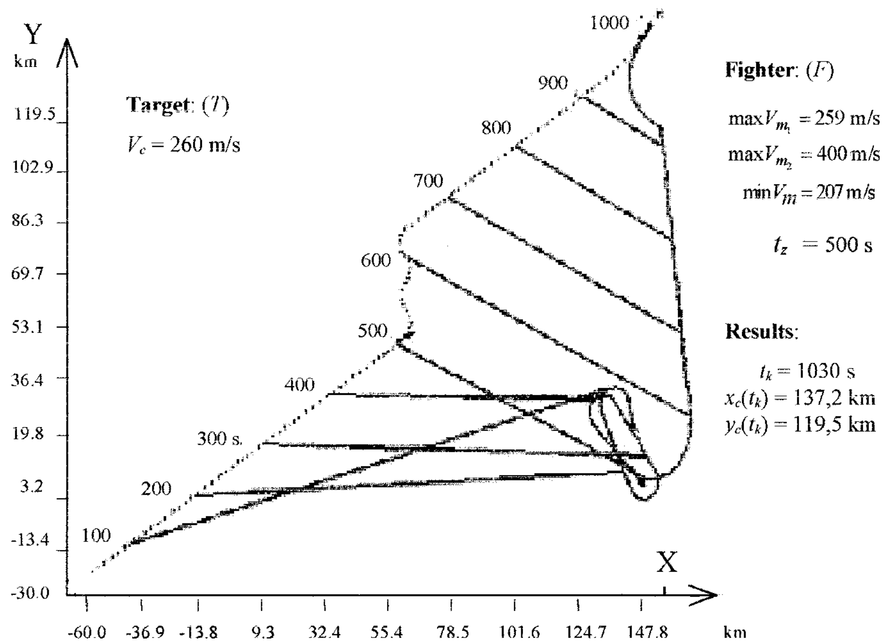

Fig. 4 Target-interception according to long-range proportional navigation guidance.

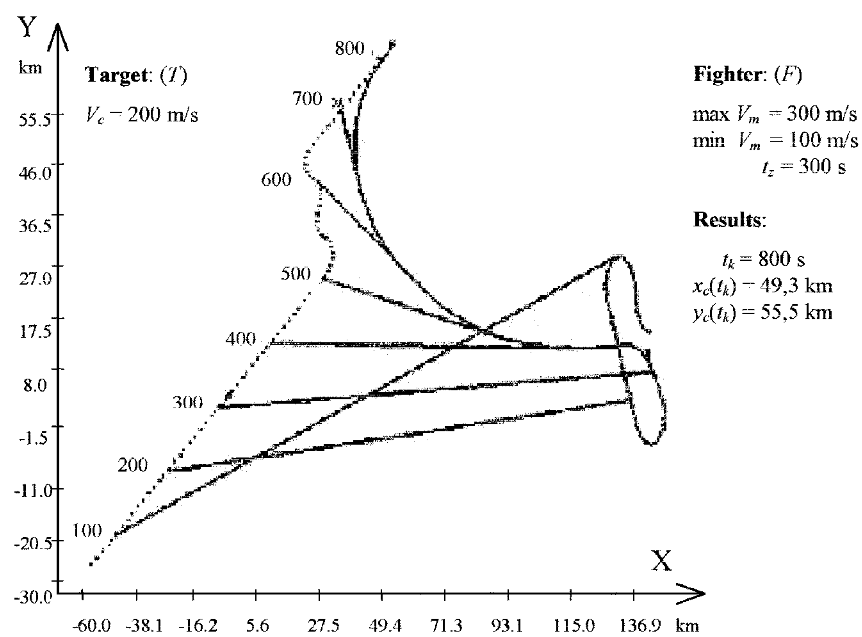

Fig. 5 Target-interception according to tail-chase on (x, y) plane.

in the time moment $t_z = 500$ s). In the preceding cases the fighter was disposed to use (during the final period of interception) his own board radar system for final guiding, thereby realizing the tail-chase trajectory on the short range. Another situation is presented in Fig. 5, where the fighter F was disposed (by the ground radar tracking and guiding system) to pursue the target in the time instant $t_z = 300$ s. In this case the fighter followed the target T according to tail-chase trajectory in the whole period of interception, and only ground radar measurements of the aircraft's positions (both the target and fighter) were needed for generation of the fighter's trajectory. Further, the much more complicated configuration is illustrated in Fig. 6. The air target T flew along his straight-line trajectory directly to the fighter F (up to the time moment $t_z = 300$ s only). In this moment the fighter F got the order (from the ground radar tracking and guiding system) to intercept the target T. Accordingly, the fighter F was automatically turned to the target T, and in the next step it was guided to the frontal attack. Numerical simulation of all of these cases shows that the proposed procedures of target interception may be realized automatically, using for this purpose the ground radar measurements of the aircraft's positions (both the target and fighter) only.

The related numerical data characterizing these scenarios are specified in the Table 1. During simulation of the cases of target interception just mentioned, the L_1 estimation has been used for succeeding aircraft positions identification because of the following reasons:

1) The mathematical model of aircraft in the form of the flat system (proposed for target tracking and fighter guiding purposes) disclosed many singular perturbations. These singularities inherently contaminated the computing results followed as a result of the execution of the procedures of target identification. An example of such kind of numerical errors (i.e., disturbances $[E]$ of target positions in sense of L_1 norm) is illustrated in Fig. 7, presenting the result of target tracking when the target maneuvered with acceleration of 4 g during realization of horizontal helical trajectory in the time period from 190 to 520 s at the average altitude of about 10,000 m and velocity of 454 m/s. The singularities appeared during the numerical identification of target maneuvers and took the form of occasional wild points or spiky noise and produced heavy-tailed, time-correlated (or colored) non-Gaussian disturbances. It is well known that this kind of low-frequency effect is similar to the low

Table 1 Initial and final state conditions of interception

No.	v_c, m/s	$v_{m_1}^{\max}$, m/s	$v_{m_1}^{\min}$, m/s	$v_{m_2}^{\max}$, m/s	$v_{m_2}^{\min}$, m/s	μ_m	ψ_m, deg	d_{rad}, m	d_r^m, m	t_z, s	t_k, s	$t_k - t_z$, s	x_c^k, km	y_c^k, km
1	200	200	100	300	100	0,5	60	30,000	5,000	200	930	730	71,3	69,2
2	260	259	207	400	207	0,5	0	30,000	5,000	500	1,030	530	137,2	119,5
3	200	300	300	300	100	0,5	0	30,000	5,000	300	800	500	49,3	55,5
4	260	259	207	400	207	0,7	60	30,000	5,000	300	340	340	83,3	61,2

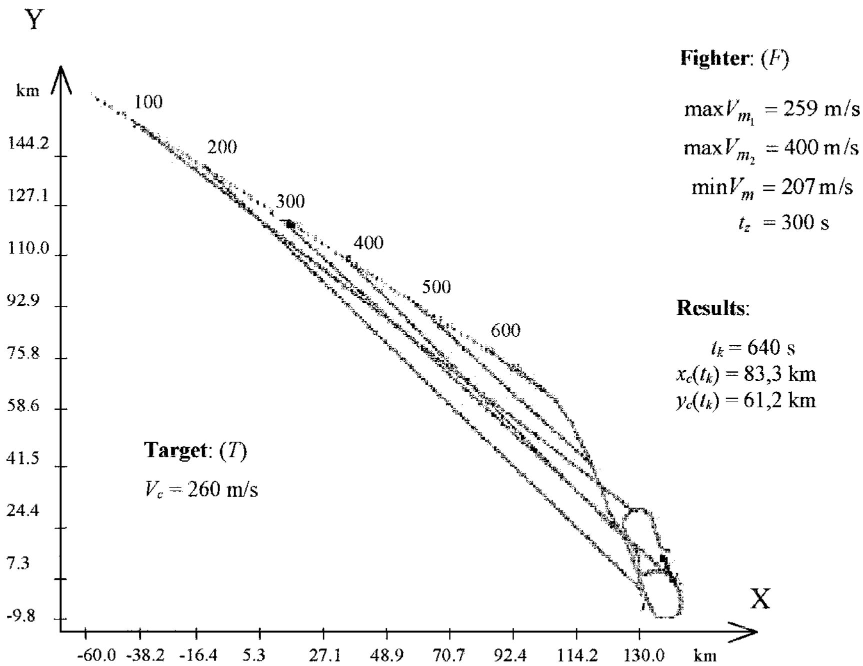

Fig. 6 Target-interception according to frontal attack on (x, y) plane.

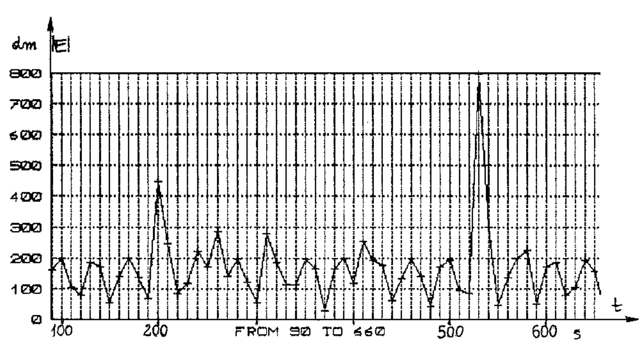

Fig. 7 Estimation of target position according to L_1 norm (i.e., absolute error).

frequency of glint noise, and it may not be possible to distinguish between actual maneuvering target motion and glint errors in radar measurements of aircraft positions.

2) The observation of these singular perturbations gives me the opportunity to use this numerical occurrence as a natural source of the chaotic disturbances especially for examination of robustness of the proposed method. For this reason the additional generation of external disturbances (in the form of spiky noise or wild points in simulated radar measurements of aircraft's positions) was not necessary during simulation experiments. Despite the influence of these numerical errors (disturbances), the simulation of target interception followed correctly and successfully.

These examples of numerical simulation indicate that the TDNN procedure developed for the differentiation of the time series of succeeding aircraft positions worked sufficiently well (without amplification of numerical errors caused by digitization of input data and singular perturbations that appeared in flat system). It can be successfully used for efficient realization of numerical procedures for aircraft trajectory tracking and target interception even against the round-off errors generated during data computing (caused by finite word length).

It was possible because both aircraft positions and their several time derivatives were smoothly reconstructed (on the basis of their discrete readings) according to the parallel procedure of diffeomorphical approximation (i.e., the quantum computation of gradients in a sense of least absolute deviation according to the L_1 norm). Because the numerical differentiation was developed simply as an algebraic inversion of the symplectic integration, high-speed computation, robustness to noises (errors), and low-cost reconstruction of signals were assigned. It means that the proposed method of filtering and smoothing the aircraft's trajectory (according to estimation in a sense of the L_1 norm) is sufficiently correct and robust to the chaotic spiky noise.

All of these aspects of numerical integration of differential equations by symmetric composition methods have been also considered lately in physics.[20–23] But it seems that the proposed TDNN structure can be fruitfully exploited in real-time applications if this scheme were considerably simplified and realized in VLSI technology. The form of the bank of differentiating filters for recognizing various classes of signals (not only the trajectory) is suggested.

Conclusions

A coordinated high-rate turn realized with modern tactical aircraft's or missiles could not be approximated by a ramp motion, by random white noise acceleration, or by random jerk motion in the state space. For these dynamic maneuvering tracks the classic theories lead to highly biased or incorrect solutions. A full set of nonlinear kinematic equations combined with flow equations of the velocity vector field of the maneuvering aircraft was used to formulate the original nonlinear tracking problem for a time-varying flat system in phase space. The TDNN was proposed for the differentiation and smoothing of an aircraft's three-dimensional positions in order to identify the translational and rotational movements of aircraft. In this proposition the differentiation was realized as an algebraic inversion of symplectic integration, and it guarantees inherent smoothing of the noisy data. The proposed TDNN structure consists of universal approximation for smooth function. Its main advantage includes avoiding the curse of dimensionality and the local minima problem. This structure presents the practical implementation of the quantum computational philosophy for air targets tracking, and it can be easily applied in real-time solutions when it can be realized in VLSI technology.

Acknowledgments

Although the Polish artificial knowledge method of differentiation was invented in 1972, its first application (1978–1980) for solving the hyperbolic partial differential equations could not be possible without the support of J. Pulaczewski and B. Beliczyński at the University of Technology in Warsaw. I am very grateful to my colleagues R. Bedyński and K. Holubowicz for their help in programming work provided during this first development. The solving of the target interception (with the use of this method of differentiation) was developed in 1991–1994 under the direction of E. Kolodziński, the head of the Institute of Automation of Command Systems at the Military University of Technology. I would like to thank E. Kowalska, T. Pietkiewicz, and K. Płomiński for their support in programming work, as well as several other of my friends for helpful discussions and input to the final form of this paper.

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